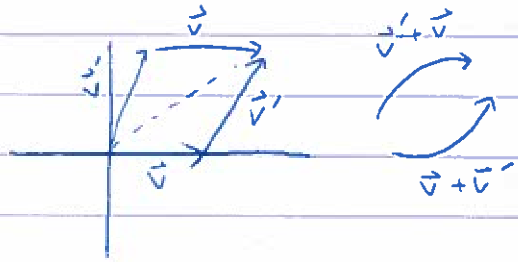
Inleveropgaven 15 feb (ma)

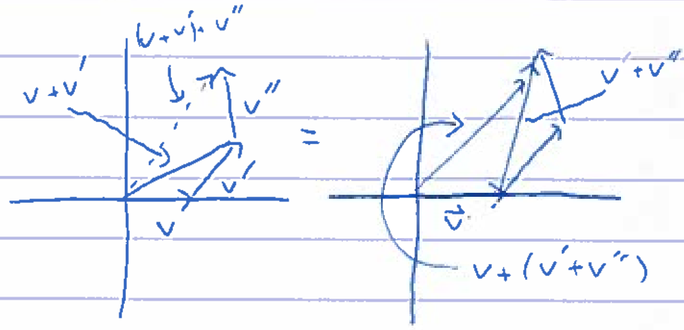
Op. 1.1 · $\vec v + \vec v' = \vec v' + \vec v$



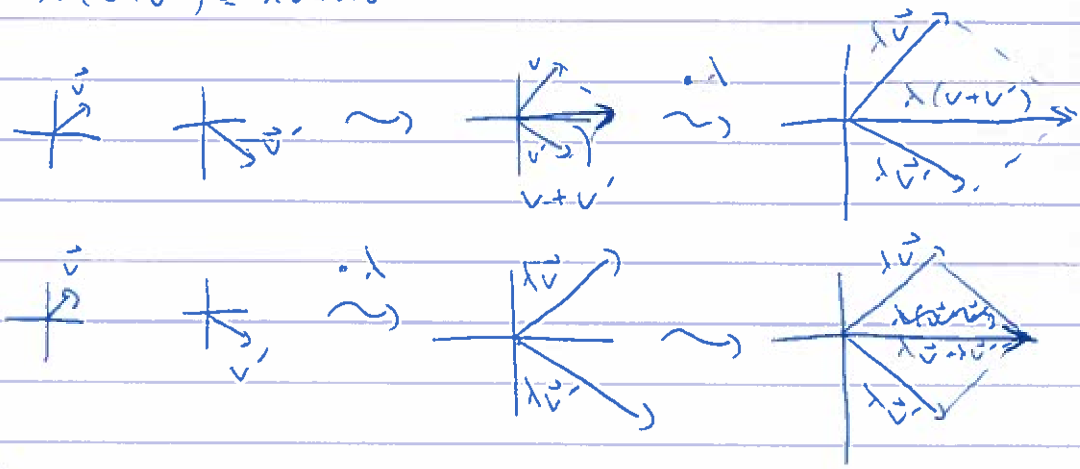
· $\vec v + (\vec v' + \vec v'')$

$=$

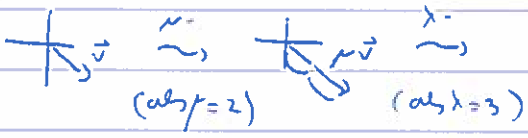
$(\vec v + \vec v') + \vec v''$

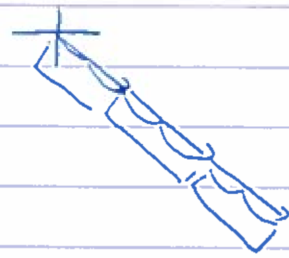


· $\lambda(v+v') = \lambda v + \lambda v'$



En voor V3
$(\lambda\mu)\vec v = \lambda(\mu v)$





En met $\lambda\mu = 6$:

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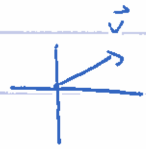
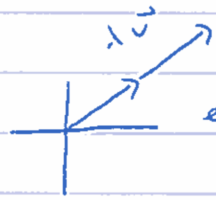
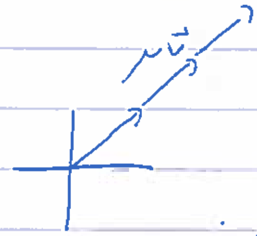
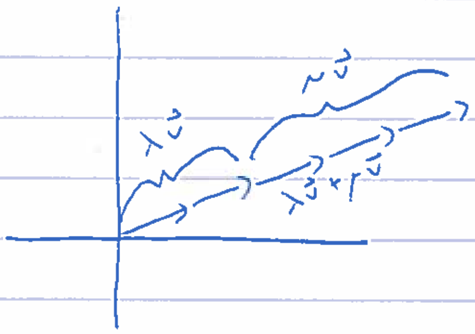
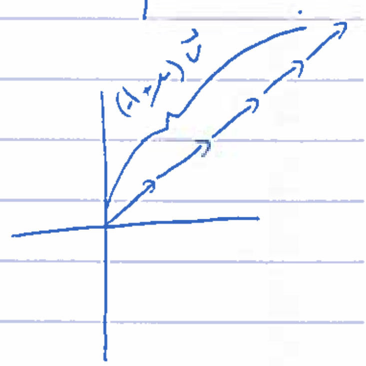
$(\lambda+\mu)\vec{v} = \lambda\vec{v} + \mu\vec{v}$

met $\lambda = 2$ en $\mu = 3$, 

geeft 

en 

Dus 

En ook $\lambda+\mu=5$, 

1.2 als $\vec{v} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ en $\vec{v}\,' = \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix}$, dan is

- $\vec{v} + \vec{v}\,' = \begin{pmatrix} x+x' \\ y+y' \\ z+z' \end{pmatrix} = \begin{pmatrix} x'+x \\ y'+y \\ z'+z \end{pmatrix} = \vec{v}\,' + \vec{v}$

met $\vec{v}\,'' = \begin{pmatrix} x'' \\ y'' \\ z'' \end{pmatrix}$ is ook

- $v + (v' + v'') = \vec{v} + \begin{pmatrix} x'+x'' \\ y'+y'' \\ z'+z'' \end{pmatrix} = \begin{pmatrix} x+x'+x'' \\ y+y'+y'' \\ z+z'+z'' \end{pmatrix}$

En ook

$(v+v') + v'' = \begin{pmatrix} x+x' \\ y+y' \\ z+z' \end{pmatrix} + v'' = \begin{pmatrix} x+x'+x'' \\ y+y'+y'' \\ z+z'+z'' \end{pmatrix}$

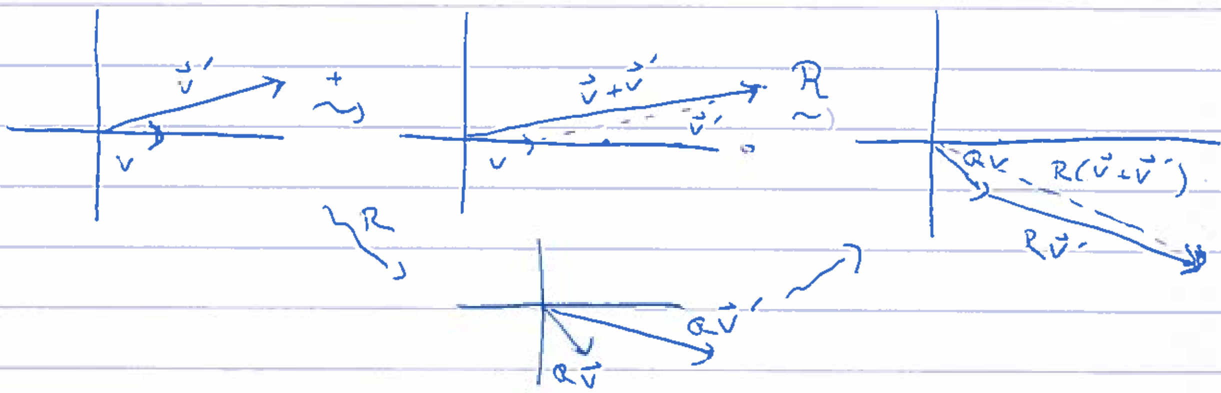
- $\lambda(v+v') = \lambda\begin{pmatrix} x+x' \\ y+y' \\ z+z' \end{pmatrix} = \begin{pmatrix} \lambda(x+x') \\ \lambda(y+y') \\ \lambda(z+z') \end{pmatrix} = \begin{pmatrix} \lambda x+\lambda x' \\ \lambda y+\lambda y' \\ \lambda z+\lambda z' \end{pmatrix}$

en $\lambda v + \lambda v' = \begin{pmatrix} \lambda x \\ \lambda y \\ \lambda z \end{pmatrix} + \begin{pmatrix} \lambda x' \\ \lambda y' \\ \lambda z' \end{pmatrix} = \begin{pmatrix} \lambda x+\lambda x' \\ \lambda y+\lambda y' \\ \lambda z+\lambda z' \end{pmatrix}$

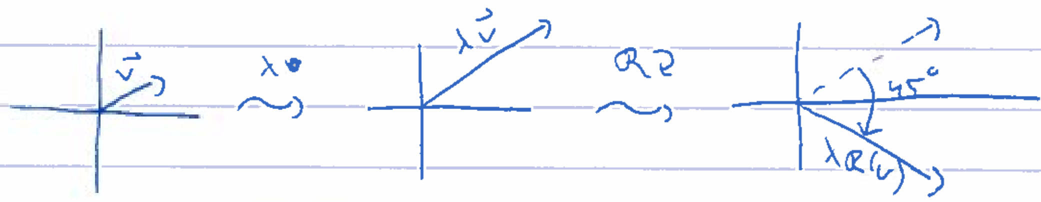
geeft hetzelfde.

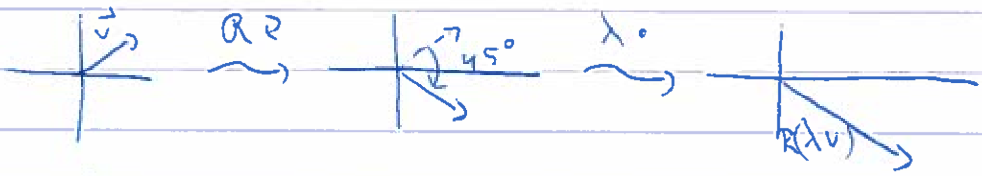
1.4 Als R een rotatie om $\pi/4$ is (ik doe ↻), dan

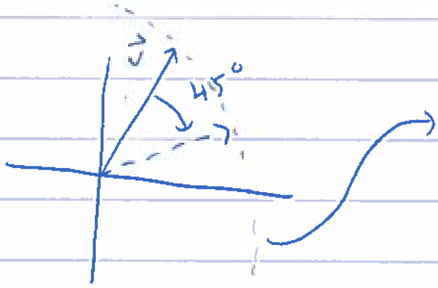
- $R(v+v') = Rv + Rv'$:



- $R(\lambda\vec{v}) = \lambda R(\vec{v})$





- 

cirkel met straal $\|\vec{v}\|$.

1.8 

Voor $\vec{v} = (0, 1, 1/\sqrt{2})$ en $\vec{v}' = (0, -1, 1/\sqrt{2})$

$(\vec{v}, \vec{v}') = -1 + 1/2 = -1/2$

$(v\,v) = 1 + 1/2 = 3/2$

$(v'v') = 1 + 1/2 = 3/2$

Dus $\cos\theta = -\dfrac{1/2}{\sqrt{3/2}\sqrt{3/2}} = -\dfrac{1/2}{3/2} = -1/3$

En $\theta = \mathrm{acos}(-1/3) = 109{,}5°$

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1.g a) Druk $\vec{v} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ uit als $\lambda_0 e_0 + \lambda_1 e_1 + \lambda_2 e_2$ en zoek $\lambda_0, \lambda_1, \lambda_2$.

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \lambda_0/\sqrt{2} \\ \lambda_0/\sqrt{2} \\ 0 \end{pmatrix} + \begin{pmatrix} \lambda_1/\sqrt{2} \\ -\lambda_1/\sqrt{2} \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \lambda_2 \end{pmatrix}$$

geeft $\lambda_2 = z$, $\frac{1}{\sqrt{2}}(\lambda_0 - \lambda_1) = y$, $\frac{1}{\sqrt{2}}(\lambda_0 + \lambda_1) = x$

dus $\begin{cases} \lambda_0 = (x+y)/\sqrt{2} \\ \lambda_1 = (x-y)/\sqrt{2} \\ \lambda_2 = z \end{cases}$

is eenduidig op te lossen, het is een basis.

Deze is orthonormaal, $(\vec{e}_i \vec{e}_j) = \delta_{ij}$

Alternatief: reken eerst na dat de $\vec{e}_i$ orthonormaal is.

Dan: $\lambda_0 = (e_0 \vec{v}) / \cancel{(e_0 e_0)}_{=1} = x+y/\sqrt{2}$

$\lambda_1 = (e_1 \vec{v}) / \cancel{(e_1 e_1)}_{=1} = (x-y)/\sqrt{2}$

$\lambda_3 = \frac{(e_2 \vec{v})}{\cancel{(e_2 e_2)}} = z$.

Controleer dat $\lambda_0 e_0 + \lambda_1 e_1 + \lambda_2 e_2$ inderdaad gelijk is aan $\vec{v} = (x, y, z)$.

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1.9 c Proberen op te lossen: $\vec{v} = \lambda_0 \vec{e}_0 + \lambda_1 \vec{e}_1 + \lambda_2 \vec{e}_2$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \lambda_0 \\ \lambda_0 \\ -\lambda_0 \end{pmatrix} + \begin{pmatrix} \lambda_1 \\ \lambda_1 \\ 0 \end{pmatrix} + \begin{pmatrix} \lambda_2 \\ \lambda_2 \\ \lambda_2 \end{pmatrix} = \begin{pmatrix} \lambda_0 + \lambda_1 + \lambda_2 \\ \lambda_0 + \lambda_1 + \lambda_2 \\ \lambda_2 - \lambda_0 \end{pmatrix}.$$

Dit heeft alleen een oplossing als $x = y$.

Een vector $\vec{v}$ met $x \neq y$ kan niet als een combinatie van e_0 e_1 e_2 geschreven worden, dus het is geen basis van $\mathbb{R}^3$.

1.12 met $\vec{v} = (x, y, z)$ hebben we:

als $\vec{v} = \lambda_0 \vec{e}_0 + \lambda_1 \vec{e}_1 + \lambda_2 \vec{e}_2$, dan

$\rightarrow (e_0, \vec{v}) = (e_0, \lambda_0 e_0) = \lambda_0 \cdot \|e_0\|^2 = 2\lambda_0$

$\hookrightarrow = x - y$

Dus $\lambda_0 = x - y/2$

$\rightarrow (e_1, \vec{v}) = (e_1, \lambda_1 e_1) = \lambda_1 \cdot 3$

$\hookrightarrow = x + y + z$

dus $\lambda_1 = (x + y + z)/3$

$\rightarrow (e_2, \vec{v}) = (e_2, \lambda_2 e_2) = \lambda_2 \cdot 6$

$\hookrightarrow x + y - 2z$

Dus $\lambda_2 = (x + y - 2z)/6$