Opg 1.11

$(\vec{e}_0, \vec{e}_1) = 1 - 1 + 0 = 0,$

$(\vec{e}_1, \vec{e}_2) = 1 + 1 - 2 = 0,$

$(\vec{e}_0, \vec{e}_2) = 1 - 1 + 0 = 0.$

Stel $\vec{v} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \lambda_0 e_0 + \lambda_1 e_1 + \lambda_2 e_2$, dus

$\langle e_0, \vec{v} \rangle = \lambda_0 (e_0, e_0) + \lambda_1 \cdot 0 + \lambda_2 \cdot 0$

Dus $1 = \lambda_0 \cdot 2 \leadsto \lambda_0 = 1/2$

$\langle e_1, \vec{v} \rangle = \lambda_1 (e_1, e_1)$,

Dus $1 = \lambda_1 \cdot 3 \leadsto \lambda_1 = 1/3$

$\langle e_2, \vec{v} \rangle = \lambda_2 (e_2, e_2)$ ~~zo~~,

dus $1 = \lambda_2 \cdot 6 \leadsto \lambda_2 = 1/6$

En $\frac{1}{2} \cdot \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} + \frac{1}{3} \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \frac{1}{6} \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix} = \begin{pmatrix} 1/2 + 1/3 + 1/6 \\ -1/2 + 1/3 + 1/6 \\ 1/3 - 1/3 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

zoals verwacht

Opg. 1.17

$i \cdot \begin{pmatrix} 1 \\ i \end{pmatrix} = \begin{pmatrix} i \\ -1 \end{pmatrix}, \quad 2i \cdot \begin{pmatrix} 1 \\ i \end{pmatrix} = \begin{pmatrix} 2i \\ -2 \end{pmatrix},$

$(1 + 2i)\begin{pmatrix} 1 \\ i \end{pmatrix} = \begin{pmatrix} 1 \\ i \end{pmatrix} + \begin{pmatrix} 2i \\ -2 \end{pmatrix} = \begin{pmatrix} 1 + 2i \\ -2 + i \end{pmatrix}$

Opg. 1.18

~~zo~~ $(1-i)\begin{pmatrix} 1 \\ 1+i \\ i \end{pmatrix} = \begin{pmatrix} 1-i \\ 2 \\ 1+i \end{pmatrix}$, dus

$(1-i)\vec{v} + \vec{v}' = \begin{pmatrix} 1-i \\ 2 \\ 1+i \end{pmatrix} + \begin{pmatrix} i \\ 1 \\ 2-i \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 3 \end{pmatrix}.$

Opgave: 1.15 c

Stel V is de 4-dimensionale deelruimte
$V = \{ \vec{v} = (x_0, \dots, x_4) \mid x_0 + \dots + x_4 = 0 \}$ van $\mathbb{R}^5$.

Orthogonaliseer de basis $e_0 = \begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$ $e_1 = \begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \\ 0 \end{pmatrix}$ $e_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ -1 \\ 0 \end{pmatrix}$ en $e_3 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ -1 \end{pmatrix}$.

Opl: stap 1: $e_0' = \begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$, $e_1' = \begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \\ 0 \end{pmatrix} - \frac{(-1)}{2} \begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1/2 \\ 1/2 \\ -1 \\ 0 \\ 0 \end{pmatrix}$,

dus $e_0' = \begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$, $e_1' = \begin{pmatrix} 1/2 \\ 1/2 \\ -1 \\ 0 \\ 0 \end{pmatrix}$

Stap 2: $e_0'' = e_0'$, $e_1'' = e_1'$, $e_2'' = e_2 - \left(0 \cdot e_1 + \frac{(-1)}{3/2} \begin{pmatrix} 1/2 \\ 1/2 \\ -1 \\ 0 \\ 0 \end{pmatrix} \right)$

dus $e_0'' = \begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$, $e_1'' = \begin{pmatrix} 1/2 \\ 1/2 \\ -1 \\ 0 \\ 0 \end{pmatrix}$ $e_2'' = \begin{pmatrix} 1/3 \\ 1/3 \\ 1/3 \\ -1 \\ 0 \end{pmatrix}$

stap 3: $e_0''' = e_0''$, $e_1''' = e_1''$, $e_2''' = e_2''$ en

$$e_3''' = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ -1 \end{pmatrix} - \left(0 \cdot e_0'' + 0\, e_1'' + \frac{(-1)}{4/3} \cdot \begin{pmatrix} 1/3 \\ 1/3 \\ 1/3 \\ -1 \\ 0 \end{pmatrix} \right)$$

dus: $e_0^{(3)} = \begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$, $e_1^{(3)} = \begin{pmatrix} 1/2 \\ 1/2 \\ -1 \\ 0 \\ 0 \end{pmatrix}$, $e_2^{(3)} = \begin{pmatrix} 1/3 \\ 1/3 \\ 1/3 \\ -1 \\ 0 \end{pmatrix}$, $e_3^{(3)} = \begin{pmatrix} 1/4 \\ 1/4 \\ 1/4 \\ 1/4 \\ -1 \end{pmatrix}$

Opgave: als $\vec{v} = (z_0, z_1)$, $\vec{v}\,' = (z_0', z_1')$, en $\lambda = i$

1.20 Ga na dat $\langle \lambda \vec{v}, \vec{v}\,' \rangle = \bar{\lambda} \langle \vec{v}, \vec{v}\,' \rangle$ en

$\langle \vec{v}, \lambda \vec{v}\,' \rangle = \lambda \langle \vec{v}, \vec{v}\,' \rangle$.

Opl:

$$\langle \lambda \vec{v}, \vec{v}\,' \rangle = \overline{i z_0} \cdot z_0' + \overline{i z_1} \cdot z_1' = -i\, \bar{z}_0 z_0' - i\, \bar{z}_1 z_1'$$

$$= (-i) \cdot (\bar{z}_0 z_0' + \bar{z}_1 z_1')$$

$$= (-i) \langle v\, v' \rangle$$

en

$$\langle \vec{v}, \lambda \vec{v}\,' \rangle = \bar{z}_0 (i z_0') + \bar{z}_1 \cdot (i z_1')$$

$$= i (\bar{z}_0 z_0' + \bar{z}_1 z_1') = i \langle v\, v' \rangle$$

Opgave: Bereken $\langle \vec{v}, \vec{v}\,' \rangle$ voor

1.19

a). $\vec{v} = (1, -i)$, $\vec{v}\,' = (1, i)$

b). $\vec{v} = (1, i)$, $\vec{v}\,' = (1, i)$

c). $\vec{v} = (1, -i)$, $\vec{v}\,' = (1, -i)$

Opl:

a) $\left\langle \begin{pmatrix} 1 \\ -i \end{pmatrix}, \begin{pmatrix} 1 \\ i \end{pmatrix} \right\rangle = 1 \cdot 1 + \overline{(-i)} \cdot i = 1 + i \cdot i = 0$

b) $\left\langle \begin{pmatrix} 1 \\ i \end{pmatrix} \begin{pmatrix} 1 \\ i \end{pmatrix} \right\rangle = 1 \cdot 1 + (-i) \cdot i = 1 + 1 = 2$

c) $\left\langle \begin{pmatrix} 1 \\ -i \end{pmatrix} \begin{pmatrix} 1 \\ -i \end{pmatrix} \right\rangle = 1 + (i) \cdot (-i) = 1 + 1 = 2$

Dus $\begin{pmatrix} 1 \\ -i \end{pmatrix} \perp \begin{pmatrix} 1 \\ i \end{pmatrix}$ en $\left\| \begin{pmatrix} 1 \\ i \end{pmatrix} \right\| = \left\| \begin{pmatrix} 1 \\ -i \end{pmatrix} \right\| = 2$

opg. 1.21 $\langle e_0 e_0\rangle = 2$, $\langle e_1 e_1\rangle = 2$, $\langle e_0 e_1\rangle = 0$

opg 1.22 a $\omega^0 = 1$, $\omega = e^{2\pi i/3} = \left(-\frac{1}{2} + \frac{1}{2}\sqrt{3}\, i\right)$, $\omega^2 = e^{\frac{2}{3}\cdot 2\pi i} = \left(-\frac{1}{2} - \frac{1}{3}\sqrt{3}\, i\right)$

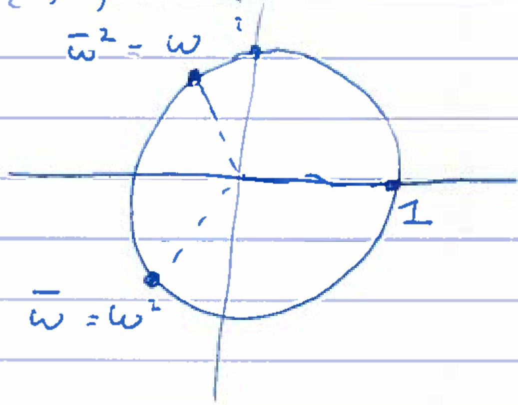
En $\omega^3 = e^{2\pi i} = 1$, dus

$$\ldots \omega^{-3}, \omega^0, \omega^3, \omega^6, \ldots = 1$$
$$\ldots \omega^{-2}, \omega^1, \omega^4, \omega^7, \ldots = e^{2\pi i/3} = \left(-\frac{1}{2} + \frac{1}{2}\sqrt{3}\, i\right)$$
$$\ldots \omega^{-1}, \omega^2, \omega^5, \omega^8, \ldots = e^{2/3(2\pi i)} = \left(-\frac{1}{2} - \frac{1}{2}\sqrt{3}\, i\right)$$

En $\bar{\omega}^3 = 1$, $\bar{\omega} = \omega^2$, $\bar{\omega}^2 = \omega$ geeft

$$\bar{\omega}^{-3}, \bar{\omega}^0, \bar{\omega}^3, \ldots = 1$$
$$\bar{\omega}^{-2}, \bar{\omega}^1, \bar{\omega}^4, \ldots = \left(-\frac{1}{2} - \frac{1}{2}\sqrt{3}\, i\right) = e^{2/3(2\pi i)}$$
$$\bar{\omega}^{-1}, \bar{\omega}^2, \bar{\omega}^5, \ldots = \left(-\frac{1}{2} + \frac{1}{2}\sqrt{3}\, i\right) = e^{2\pi i/3}$$

Merkop: $\omega^{-1} = \bar{\omega}$



b) $\langle e_0 e_0\rangle = 3$
$\langle e_1 e_1\rangle = 3$
$\langle e_2 e_2\rangle = 3$

$\langle e_0 e_1\rangle = 1 + \omega + \omega^2 = 0$ (Zie plaatje: som van 3 punten op mercedes-ster)

$\langle e_0 e_2\rangle = 1 + \omega^2 + \omega^4$
$= 1 + \omega + \omega^2 = 0$

~~$\langle e_1 e_2\rangle = 1 + \omega^3 + \omega^6 = 1+1+1$~~

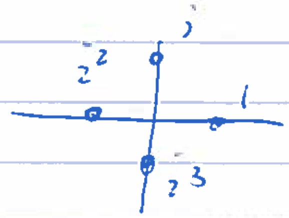
$\langle e_1, e_2\rangle = 1\cdot 1 + \bar{\omega}\cdot\omega^2 + \bar{\omega}^2\cdot\omega^4$
$= 1 + \omega^{-1}\cdot\omega^2 + \omega^{-2}\cdot\omega^4$
$= 1 + \omega + \omega^2 = 0$.

1.23 · $\langle e_0 e_0 \rangle = \langle e_1 e_1 \rangle = \langle e_2 e_2 \rangle = \langle e_3 e_3 \rangle = 4$

· $\langle e_0 e_1 \rangle = 1 + i + i^2 + i^3 = 0$

· $\langle e_0 e_2 \rangle = 1 + \cancel{i^2} - 1 + 1 - 1 = 0$

· $\langle e_0 e_3 \rangle = 1 - i - 1 + i = 0$



· $\langle e_1 e_2 \rangle = 1 + \bar{i}\, i^2 + \bar{i}^2 i^4 + \bar{i}^3 i^6$
$= 1 + i + i^2 + i^3 = 0$ (gebruik $i\bar{i} = 1$)

· $\langle e_1 e_3 \rangle = 1 + \bar{i}\, i^3 + \bar{i}^2 i^6 + \bar{i}^3 i^9$
$= 1 + i^2 + i^4 + i^6$
$= 1 - 1 + 1 - 1 = 0.$

· $\langle e_2 e_3 \rangle = 1 + i - 1 - i = 0$

1.24 : De vectoren $\vec{e}_0, \dots, \vec{e}_{n-1}$ in $\mathbb{C}^n$ met

$\vec{e}_0 = (1, \dots, 1)$

$\vec{e}_1 = (1, e^{2\pi i/n}, e^{\frac{2}{n}(2\pi i)}, \dots, e^{\frac{n-1}{n}\cdot 2\pi i})$

$\vec{e}_2 = (1, e^{\frac{2}{n}(2\pi i/n)}, e^{\frac{4}{n}(2\pi i)}, \dots, e^{2\frac{n-1}{n}\cdot 2\pi i})$

$\vdots$

$\vec{e}_k = (1, e^{k\cdot 2\pi i/n}, e^{2k(2\pi i/n)}, \dots, e^{k(n-1)(\frac{2\pi i}{n})})$

$\vdots$

$\vec{e}_{n-1} = (1, e^{(n-1)(2\pi i/n)}, \dots, e^{(n-1)^2(2\pi i/n)})$

zijn orthogonaal, met lengte $\sqrt{n}$.