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Opg. als $f + h = g$, dan $c_k + c_k' = c_k''$.

schrijf nu $f + 1 = g$.

- De c_k zijn $\frac{1}{\pi i k}\left(1 - (-1)^k\right)$ als $k \neq 0$, 0 als $k = 0$
- De c_k' zijn

$$c_k' = \begin{cases} 0 & \text{als } k \neq 0 \\ c_0' = \frac{1}{2}\int_{-1}^{1} 1 \, dx = 1 \end{cases}$$

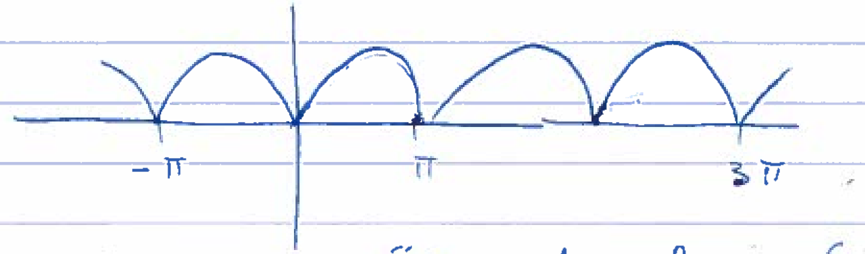
(want $1 = 1 \cdot 1 + \sum_{\substack{k=-N \\ k \neq 0}}^{N} 0 \cdot e^{2\pi i k x/2}$)

Dus

$$c_k'' = c_k + c_k' = \begin{cases} \frac{1}{\pi i k}\left(1 - (-1)^k\right) & \text{als } k \neq 0 \\ 1 & \text{als } k = 0 \end{cases}$$

OGG:

a)



f is even en reëel, dus $b_k = 0$ (geen sinussen)

en a_k is reëel

b) $$b_k = \frac{1}{\pi}\int_{-\pi}^{\pi} |\sin(x)|\sin(kx)\,dx = 0 \quad \text{(oneven functie!)}$$

$$a_k = \frac{1}{\pi}\int_{-\pi}^{\pi} |\sin(x)|\cos(kx)\,dx$$

$$= \frac{1}{\pi}\cdot 2 \cdot \int_0^{\pi} \sin(x)\cos(kx)\,dx$$

$$= \frac{2}{\pi}\int_0^{\pi} \frac{e^{ix}-e^{-ix}}{2i}\cdot\frac{e^{ikx}+e^{-ikx}}{2}$$

$$= \frac{1}{2\pi i}\int_0^{\pi} e^{i(k+1)x} - e^{i(k-1)x} + e^{-i(k-1)x} - e^{-i(k+1)x}\,dx$$

$$= \frac{1}{\pi}\cdot\int_0^{\pi} \sin((k+1)x) - \sin((k-1)x)\,dx$$

$$= \frac{1}{\pi}\left(\left[-\frac{1}{k+1}\cos((k+1)x)\right]_0^{\pi} + \left[\frac{1}{k-1}\cos((k-1)x)\right]_0^{\pi}\right)$$

als $k \neq 1$ of $k \neq -1$

Dus: $k \neq 1$, dan ~, $a_k = \frac{1}{\pi}\left(-\frac{1}{k+1}\left((-1)^{k+1}-1\right) + \frac{1}{k-1}\left(\overbrace{(-1)^{k-1}}^{=(-1)^{k+1}}-1\right)\right)$

$$a_k = \frac{(-1)^{k+1}-1}{\pi}\cdot\left(\frac{1}{k-1}-\frac{1}{k+1}\right) = \frac{(-1)^{k+1}-1}{\pi}\cdot\frac{2}{k^2-1}$$

want $\frac{1}{k-1}-\frac{1}{k+1} = \frac{k+1}{(k-1)(k+1)} - \frac{(k-1)}{(k+1)(k-1)} = \frac{2}{k^2-1}$.

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Als $k=1$, dan

$$a_1 = \frac{2}{\pi}\int_0^{\pi} \sin(x)\cos(x)\,dx = \frac{1}{\pi}\int_0^{\pi} \sin(2x)\,dx$$

$$= \frac{1}{\pi}\cdot\left[-\frac{1}{2}\cos(2x)\right]_0^{\pi} = 0$$

~~Met~~ $\cancel{a_1 = \frac{2}{\pi}\int_0^{\pi} \sin}$

Dus: $b_k = 0$, $a_k = \begin{cases} 0 & \text{als } k=1 \\ \frac{2}{\pi}\,\frac{(-1)^{k+1}-1}{k^2-1} & \text{als } k \neq 1 \end{cases}$

$\left(a_0 = \frac{4}{\pi},\ a_1 = 0,\ a_2 = \frac{-4}{3\pi},\ a_3 = 0,\ a_4 = \frac{-4}{15\pi},\ a_5 = 0,\ \ldots\right)$

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op $[-\pi, \pi)$

als $f(x) = e^{ax}$, dan zijn de fouriercoëfficiënten

$$c_k = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{ax} e^{-ikx}\,dx = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{(a-ik)x}\,dx$$

$$= \frac{1}{2\pi}\left[\frac{1}{a-ik} e^{(a-ik)x}\right]_{-\pi}^{\pi} = \frac{1}{2\pi(a-ik)}\left((-1)^k e^{a\pi} - (-1)^k e^{-a\pi}\right)$$

$$= \frac{e^{a\pi} - e^{-a\pi}}{2\pi} \cdot \frac{(-1)^k}{a-ik}$$

Voor de Stelling van Parseval:

- $$\frac{1}{2\pi}\int_{-\pi}^{\pi} |f|^2\,dx = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{2ax}\,dx = \frac{1}{2\pi}\cdot\frac{1}{2a}\left[e^{2ax}\right]_{-\pi}^{\pi}$$

$$= \frac{1}{4a\pi}\left(e^{2a\pi} - e^{-2a\pi}\right)$$

- $$\lim_{N\to\infty}\sum_{k=-N}^{N} |c_k|^2 = \lim_{N\to\infty}\sum_{k=-N}^{N}\left(\frac{e^{a\pi}-e^{-a\pi}}{2\pi}\right)^2 \cdot \frac{1}{a^2+k^2}$$

$$= \left(\frac{e^{a\pi}-e^{-a\pi}}{2\pi}\right)^2 \cdot \left(\frac{1}{a^2} + 2\sum_{k=1}^{\infty}\frac{1}{a^2+k^2}\right)$$

Dus $$\frac{1}{4a\pi}\left(e^{2a\pi} - e^{-2a\pi}\right) = \left(\frac{e^{a\pi}-e^{-a\pi}}{2\pi}\right)^2\left(\frac{1}{a^2} + 2\sum_{k=1}^{\infty}\frac{1}{a^2+k^2}\right)$$

$$\frac{\pi}{a}\,\frac{e^{2a\pi}-e^{-2a\pi}}{(e^{a\pi}-e^{-a\pi})^2} = \frac{1}{a^2} + 2\sum_{k=1}^{\infty}\frac{1}{a^2+k^2}$$

$$\sum_{k=1}^{\infty}\frac{1}{a^2+k^2} = \frac{\pi}{2a}\,\frac{e^{2a\pi}-e^{-2a\pi}}{(e^{a\pi}-e^{-a\pi})^2} - \frac{1}{2a^2}$$