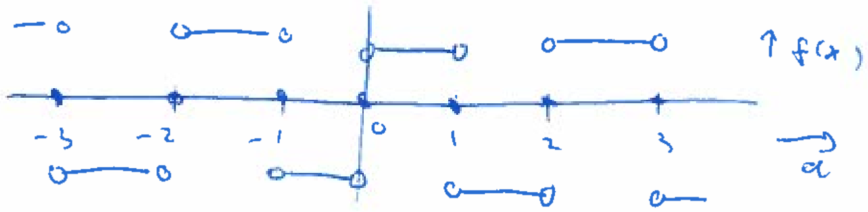
WC Ma
22 feb
1/2

opg $f(x) = -1$ op $(-1,0)$, $f(x) = 1$ op $(0,1)$,
$f(0) = f(1) = 0$, en f is 2-periodiek



a) De functie is reëel en oneven, dus $\bar{c}_k = c_{-k}$ en $c_{-k} = -c_k$,
dus $\bar{c}_k = -c_k$ en $c_{-k} = -c_k$
↳ c_k is zuiver imaginair.

b)
$$c_k = \frac{1}{2} \cdot \int_{-1}^{1} f(x)\, e^{-2\pi i k x/2}\, dx$$
$$= \frac{1}{2} \cdot \int_{-1}^{0} (-1)\, e^{-\pi i k x}\, dx + \int_{0}^{1} (+1)\, e^{-\pi i k x}\, dx$$
$$= \frac{1}{2} \cdot \left(\left[\frac{1}{\pi i k} e^{-\pi i k x} \right]_{-1}^{0} + \left[\frac{-1}{\pi i k} e^{-\pi i k x} \right]_{0}^{1} \right)$$
$$= \frac{1}{2\pi i k} \left(\left(1 - (-1)^k\right) + \left(-(-1)^k + 1 \right) \right)$$
$$= \frac{1}{\pi i k} \left(1 - (-1)^k \right)$$

opg c Fourier coëfficiënten van $f(x) = \sin x$

Antw: $\sin(x) = \frac{1}{2i} \left(e^{ix} - e^{-ix} \right)$.

Dus $f(x) = c_1 e^{ix} + c_{-1} e^{-ix}$ met $c_1 = \frac{1}{2i}$, $c_{-1} = \frac{-1}{2i}$.

De overige Fourier coëfficiënten zijn nul.

opg:

WCMa
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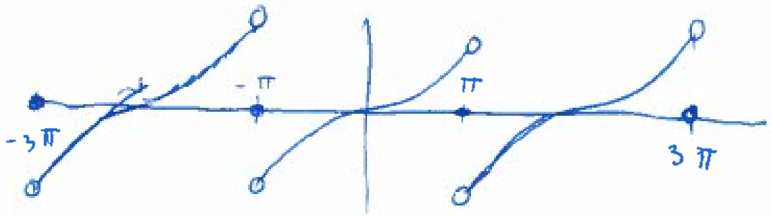
$2/\frac{2}{2}$



$f(x) = \frac{e^x - e^{-x}}{2}$ als $|x| < \pi$,

$f(\pi) = 0$.

a) f is oneven en reëel, dus $\bar{c}_k = c_{-k}$ ~~MAX~~ ; en $c_{-k} = -c_k$

dus $-c_k = \bar{c}_k$ en c_k is ~~reëel~~, zuiver imaginair.

b) $$c_k = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{e^x - e^{-x}}{2} e^{-ikx}\,dx = \frac{1}{4\pi}\left(\int_{-\pi}^{\pi} e^{x-ikx}\,dx - \int_{-\pi}^{\pi} e^{-x-ikx}\,dx\right)$$

$$= \frac{1}{4\pi}\cdot\left(\left[\frac{1}{1-ik}e^{(1-ik)x}\right]_{-\pi}^{\pi} - \left[\frac{1}{-1-ik}e^{(-1-ik)x}\right]_{-\pi}^{\pi}\right)$$

$$= \frac{1}{4\pi}\cdot\left(\frac{1}{1-ik}\left((-1)^k e^{\pi} - (-1)^k e^{-\pi}\right) - \frac{1}{-1-ik}\left((-1)^k e^{-\pi} - (-1)^k e^{\pi}\right)\right)$$

$$= \frac{(-1)^k(e^{\pi} - e^{-\pi})}{4\pi}\left(\frac{1}{1-ik} - \frac{1}{1+ik}\right)$$

$$= \frac{(-1)^k(e^{\pi} - e^{-\pi})}{4\pi}\left(\frac{1+ik}{(1-ik)(1+ik)} - \frac{1-ik}{(1-ik)(1+ik)}\right)$$

$$= \frac{e^{\pi} - e^{-\pi}}{4\pi}\cdot\frac{2i(-1)^k\cdot k}{1+k^2}$$

Dus $$P_N(x) = \sum_{k=-N}^{N} c_k e^{ikx}$$

$$= c_0 + \sum_{k=1}^{N} c_k e^{ikx} + c_{-k}e^{-ikx} =$$ want $c_{-k} = -c_k$

$$= \underbrace{c_0}_{=0} + \sum_{k=1}^{N} c_k\left(e^{ikx} - e^{-ikx}\right)$$

$$= \frac{e^{\pi} - e^{-\pi}}{2\pi}\sum_{k=1}^{N}\frac{(-1)^k\cdot k}{1+k^2}\cdot i\,\underbrace{\left(e^{ikx} - e^{-ikx}\right)}_{=2i\sin(kx)}$$

$$= \frac{e^{\pi} - e^{-\pi}}{\pi}\sum_{k=1}^{N}\frac{(-1)^{k+1}\cdot k}{1+k^2}\sin(kx)$$