Uitwerkingen Deeltentamen A

(1) a) $\|\vec{v}\| = \sqrt{\langle v, v\rangle} = \sqrt{1^2+0^2+1^2} = \sqrt{2}$

$\|\vec{v}'\| = \sqrt{\langle v' v'\rangle} = \sqrt{0^2+1^2+1^2} = \sqrt{2}$

$\Theta = \arccos\left(\frac{\langle v, v'\rangle}{\|v\|\|v'\|}\right) = \arccos\left(\frac{1}{2}\right) = \pi/3 \; (60^\circ)$

b) als $w = a\vec{v} + b\vec{v}'$ in W zit, dan zit ook $\lambda w = (\lambda a)\vec{v} + (\lambda b)v'$ in W voor elke λ in $\mathbb{R}$.
als $w_1 = a_1\vec{v} + b_1\vec{v}'$ en $w_2 = a_2\vec{v} + b_2\vec{v}'$ beide in W zitten, dan zit ook $w_1 + w_2 = (a_1+a_2)\vec{v} + (b_1+b_1')\vec{v}'$ in W. Ook zit $0 = 0\vec{v} + 0w'$ in W.

c) $\vec{b}_0' = b_0$, $\vec{b}_1' = b_1 - \frac{\langle b_0, b_1\rangle}{\langle b_0, b_0\rangle}\vec{b}_0 = \vec{b}_1 - \frac{1}{2}\vec{b}_0$

Dus $\vec{b}_0' = \begin{pmatrix}1\\0\\1\end{pmatrix}$ $\vec{b}_1' = \begin{pmatrix}0\\1\\1\end{pmatrix} - \frac{1}{2}\begin{pmatrix}1\\0\\1\end{pmatrix} = \begin{pmatrix}-1/2\\1\\1/2\end{pmatrix}$.

d) $P = \lambda_0\vec{b}_0' + \lambda_1\vec{b}_1'$ met $\lambda_0 = \left\langle \begin{pmatrix}1\\0\\1\end{pmatrix} \middle| \begin{pmatrix}0\\0\\3\end{pmatrix}\right\rangle \Big/ \left\langle \begin{pmatrix}1\\0\\1\end{pmatrix}, \begin{pmatrix}1\\0\\1\end{pmatrix}\right\rangle = 3/2$

en $\lambda_1 = \left(\begin{pmatrix}-1/2\\1\\1/2\end{pmatrix} \middle| \begin{pmatrix}0\\0\\3\end{pmatrix}\right) \Big/ \left(\begin{pmatrix}-1/2\\1\\1/2\end{pmatrix} \middle| \begin{pmatrix}-1/2\\1\\1/2\end{pmatrix}\right) = \frac{3/2}{1+\frac{1}{4}+\frac{1}{4}} = 1$

Dus de projectie van $\begin{pmatrix}0\\0\\3\end{pmatrix}$ op W is

$\vec{p} = \frac{3}{2}\cdot\begin{pmatrix}1\\0\\1\end{pmatrix} + 1\cdot\begin{pmatrix}-1/2\\1\\1/2\end{pmatrix} = \begin{pmatrix}1\\1\\2\end{pmatrix}$

② a) $b_0' = b_0$, $\quad b_1' = b_1 - \frac{\langle b_0 b_1 \rangle}{\langle b_0 b_0 \rangle} b_0$ met

$\langle b_0 b_1 \rangle = \int_0^1 x^2\,dx = \frac{1}{3}$

$\langle b_0 b_0 \rangle = \int_0^1 x\,dx = \frac{1}{2}$ $\Big\}\ \frac{\langle b_0 b_1 \rangle}{\langle b_0 b_0 \rangle} = \frac{2}{3}$

Dus $b_0'(x) = \sqrt{x}$, $b_1'(x) = x\sqrt{x} - \frac{2}{3}\sqrt{x}$

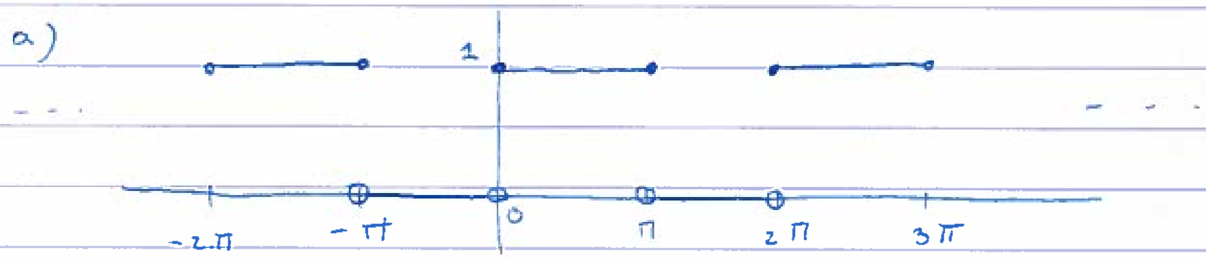
b) $\|b_0'\|^2 = \int_0^1 x\,dx = \frac{1}{2}$, dus $\|b_0'\| = \frac{1}{2}\sqrt{2}$

$\|b_1'\|^2 = \int_0^1 (x\sqrt{x} - \frac{2}{3}\sqrt{x})^2\,dx = \int_0^1 x^3 - \frac{4}{3}x^2 + \frac{4}{9}x\,dx$

$= \frac{1}{4} - \frac{4}{9} + \frac{2}{9} = \frac{1}{4} - \frac{2}{9} = \frac{9-8}{36} = \frac{1}{36}$.

Dus $\|b_0'\| = \frac{1}{2}\sqrt{2}$, $\|b_1'\| = 1/6$.

③ a)



b) $a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx = \frac{1}{\pi}\cdot\pi = 1$

voor $k = 1, 2, \ldots$ $\quad b_k = \frac{1}{\pi}\int_0^{\pi} \sin(kx)\,dx = \frac{1}{\pi}\left[-\frac{1}{k}\cos(kx)\right]_0^{\pi}$

$= \frac{1}{\pi}\left(-\frac{1}{k}\cos(k\pi) + \frac{1}{k}\right) = \frac{1}{\pi k}\left(1 - (-1)^k\right)$

voor $k = 1, 2, \ldots$ $\quad a_k = \frac{1}{\pi}\int_0^{\pi} \cos(kx)\,dx = \frac{1}{\pi}\left[\frac{1}{k}\sin(kx)\right]_0^{\pi} = 0$.

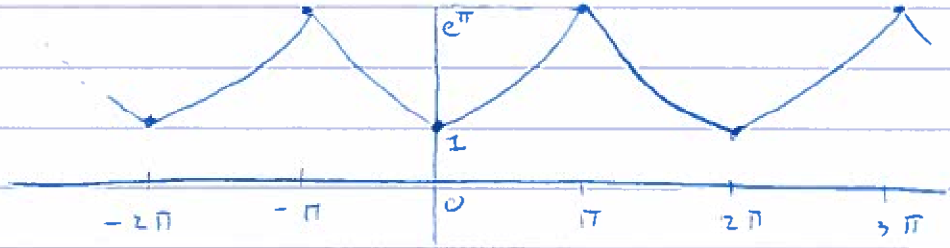
Dus $b_k = \frac{1}{\pi k}\left(1 - (-1)^k\right)$ voor $k > 0$, $a_0 = 1$,

en $a_k = 0$ voor $k > 0$

c) $\lim_{N \to \infty} \left(\frac{1}{2} + \sum_{k=1}^{N} \overbrace{a_k}^{=0} \cos(k \cdot 0) + \sum_{k=1}^{N} b_k \overbrace{\sin(k \cdot 0)}^{=0}\right) = \frac{1}{2}$

of: $\lim_{N \to \infty} P_N(x) = \frac{1}{2}\left(f(0_+) + f(0_-)\right) = \frac{1}{2}(1 + 0) = \frac{1}{2}$.

④ a)



b) $$c_k = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{|x|} e^{-ikx}\,dx = \frac{1}{2\pi}\left(\int_{-\pi}^{0} e^{-x} e^{-ikx}\,dx + \int_{0}^{\pi} e^{x} e^{-ikx}\,dx\right)$$

$$= \frac{1}{2\pi}\left(\int_{0}^{\pi} e^{(1-ik)x}\,dx + \int_{-\pi}^{0} e^{-(1+ik)x}\,dx\right)$$

$$= \frac{1}{2\pi}\cdot\left(\left[\frac{1}{1-ik} e^{(1-ik)x}\right]_{0}^{\pi} + \left[\frac{-1}{1+ik} e^{-(1+ik)x}\right]_{-\pi}^{0}\right)$$

$$= \frac{1}{2\pi}\left(\frac{e^{\pi}\cdot(-1)^k - 1}{1-ik} + \frac{-1 + e^{\pi}(-1)^k}{1+ik}\right)$$

$$= \frac{(-1)^k e^{\pi} - 1}{2\pi}\left(\frac{1}{1-ik} + \frac{1}{1+ik}\right)$$

$$= \frac{(-1)^k e^{\pi} - 1}{2\pi}\left(\frac{(1+ik) + (1-ik)}{1+k^2}\right)$$

$$= \frac{(-1)^k e^{\pi} - 1}{\pi(1+k^2)}$$

c) $$B_1(x) = c_0 + c_1 e^{ix} + c_{-1} e^{-ix}$$

$$= \frac{(e^{\pi}-1)}{\pi} + \frac{-e^{\pi}-1}{2\pi} e^{ix} + \frac{-e^{\pi}-1}{2\pi} e^{-ix}$$

$$= \frac{e^{\pi}-1}{\pi} - \frac{(1+e^{\pi})}{2\pi}\left(e^{ix} + e^{-ix}\right)$$

$$= \frac{1}{2}\left(2\,\frac{e^{\pi}-1}{\pi}\right) - \frac{(1+e^{\pi})}{\pi}\cos(x) + 0\sin(x)$$

Dus $a_0 = 2\frac{(e^{\pi}-1)}{\pi}$, $a_1 = -\frac{(1+e^{\pi})}{\pi}$, $b_1 = 0$.