Hamiltonian complexity meets derandomization

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joint work with Dorit Aharonov

Randomness helps...

- Communication complexity
- Query complexity
- Cryptography

... in all cases?

- Under believable assumptions, randomness does not increase computational power
- It should be true, but how to prove it?

A glimpse of its hardness

Polynomial identity testing problem

Input: A representation of a polynomial $p : \mathbb{F}^n \to \mathbb{F}$ of degree $d(n)$
Output: Yes iff $\forall x_1, ..., x_n \in \mathbb{F}, p(x_1, ..., x_n) = 0$

- Simple randomized algorithm
  - Pick $x_1, ..., x_n$ uniformly at random from a finite set $S \subseteq \mathbb{F}$
  - If $p \neq 0$, $Pr[p(x_1, ..., x_n) = 0] \leq \frac{d}{|S|}$
- How to find such "witness" deterministically?

MA vs. NP

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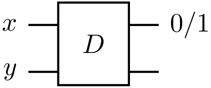
Problem $L \in \mathrm{NP}$



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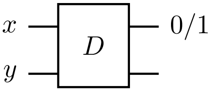

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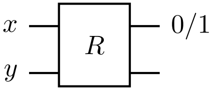

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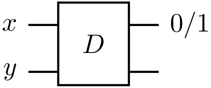

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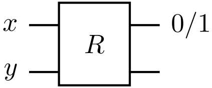

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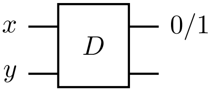

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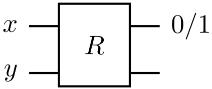

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Derandomization conjecture

$$MA = NP$$

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Local Hamiltonian problem (k-LH$_{\alpha,\beta}$)

Input: Local Hamiltonians H_1, ... H_m, each acting on k out of a n-qubit system; $H = \sum_i H_i$

yes-instance: $\langle\psi| H |\psi\rangle \leq \alpha m$ for some $|\psi\rangle$

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How hard is this problem?

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- In this work: $|\phi_{i,j}\rangle = |T_{i,j}\rangle$, where $T_{i,j} \subseteq \{0,1\}^k$

Stoquastic Hamiltonian problem

Uniform stoquastic local Hamiltonian problem

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- Our work: if β is constant, it is in NP

Outline

1. Connection between Hamiltonian complexity and derandomization
2. MA and stoquastic Hamiltonians
3. Proof sketch
4. Open problems

Back to NP vs. MA

Theorem (BT '08)

Deciding if Unif. Stoq. LH is frustration-free or inverse polynomial frustrated is MA-complete.

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1. H' *is a uniform stoquastic Hamiltonian with constant locality and degree;*
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Then $MA = NP$.

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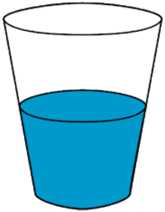

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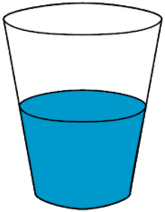

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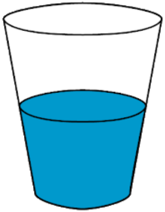

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Stoquastic Hamiltonians in MA (BT '08)

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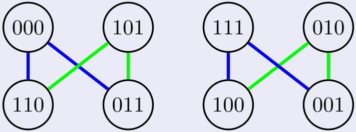

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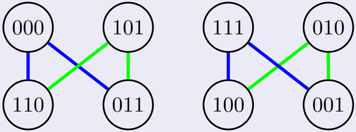

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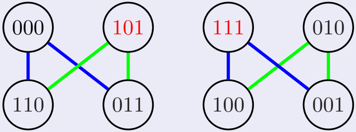

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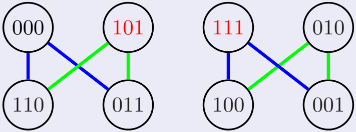

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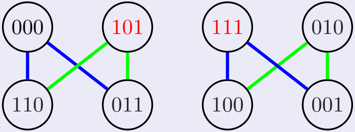

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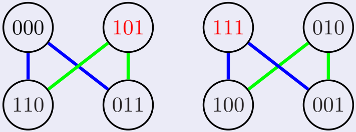

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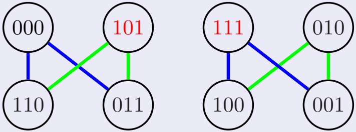

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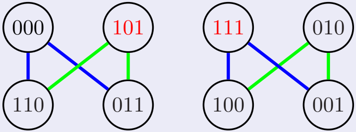

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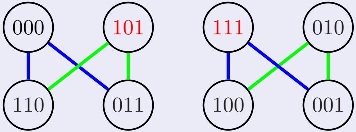

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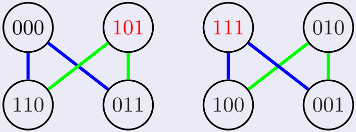

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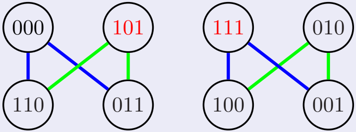

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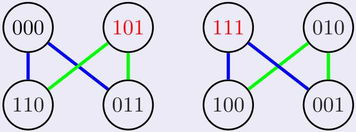

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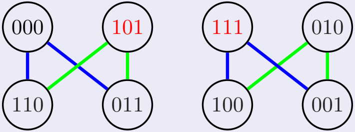

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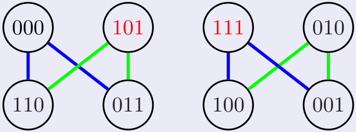

$$000 \xrightarrow{P_{1,2}} 000 \xrightarrow{P_{1,2}} 110 \xrightarrow{P_{1,3}} 110 \xrightarrow{P_{2,3}} 101$$

Stoquastic Hamiltonians in MA (BT '08)

Theorem

If H is FF and x_0 is in some groundstate of H, then the verifier never reaches a bad string.
If H is $1/poly(n)$ frustrated, then the random-walk rejects with constant probability.

Very frustrated case

Theorem

If H is εm frustrated for some constant ε, then from every initial string there is a constant-size path that leads to a bad string.

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Check if any of the constant-size paths reaches a bad string.

- For yes-instances, this is never the case (BT' 08).
- For no-instances, this is always the case (previous theorem).

$\square$

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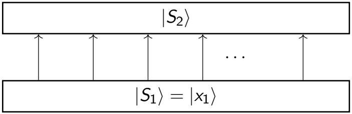
Structure of the proof

1. There is a **constant-depth** “circuit” of non-overlapping projectors that achieves state with a bad string
   1. Construct circuit layer by layer: either there is a bad string, or we can add a new layer that brings us closer to a bad string
2. From the constant-depth circuit, we can use a **lightcone**-argument to retrieve a constant-size path.

States with a bad string

$\boxed{|S_1\rangle = |x_1\rangle}$ 1 string

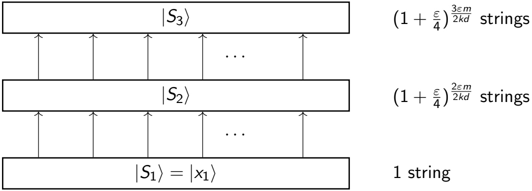
States with a bad string



$(1+\frac{\varepsilon}{4})^{\frac{2\varepsilon m}{2kd}}$ strings

1 string

States with a bad string



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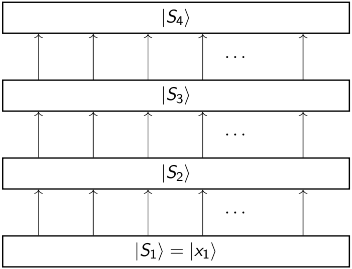

$(1+\frac{\varepsilon}{4})^{\frac{4\varepsilon m}{2kd}}$ strings

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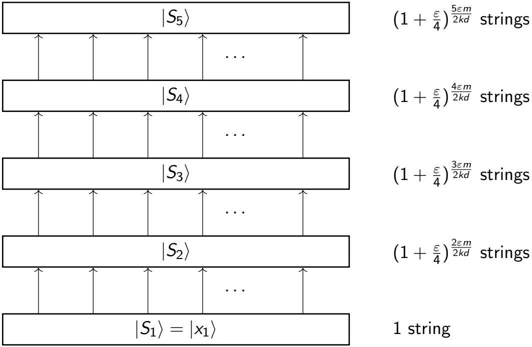

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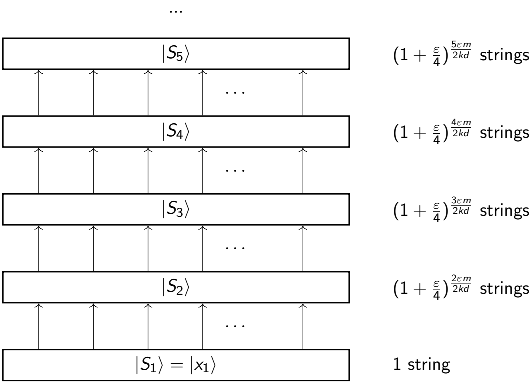

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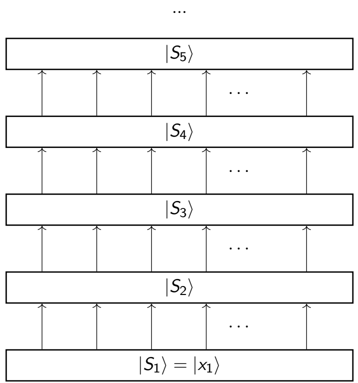

$(1+\frac{\varepsilon}{4})^{\frac{5\varepsilon m}{2kd}}$ strings

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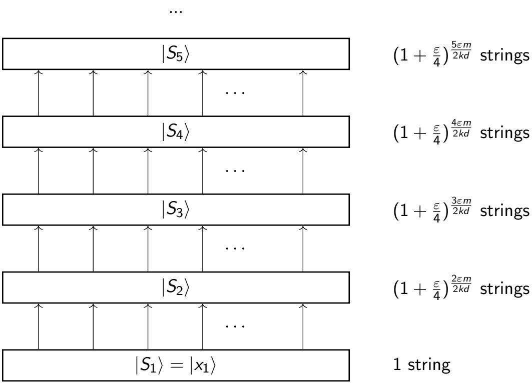
Finding a bad string

Pick $L = \frac{\varepsilon m}{2kd}$, the frustration is at least $\frac{\varepsilon}{2}$, there is a constant T such that $|S_T\rangle = |+\rangle^{\otimes n}$

States with a bad string



Finding a bad string

Pick $L = \frac{\varepsilon m}{2kd}$, the frustration is at least $\frac{\varepsilon}{2}$, there is a constant T such that $|S_T\rangle = |+\rangle^{\otimes n} \Rightarrow$ there is a bad string in $|S_T\rangle$.

From shallow non-overlapping transitions to short paths

Lemma

If a bad string is reached after a constant number of non-overlapping projections, then there is a constant-size path to a bad string.

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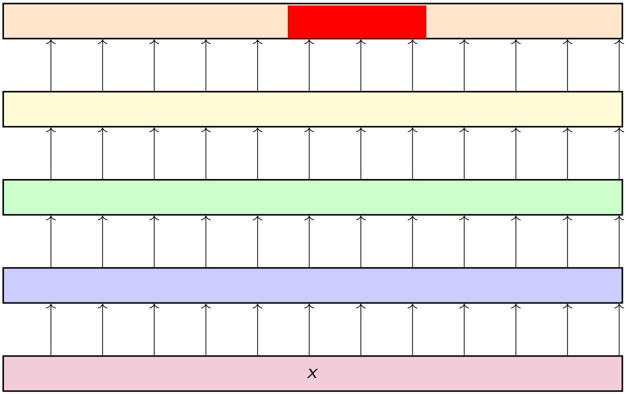

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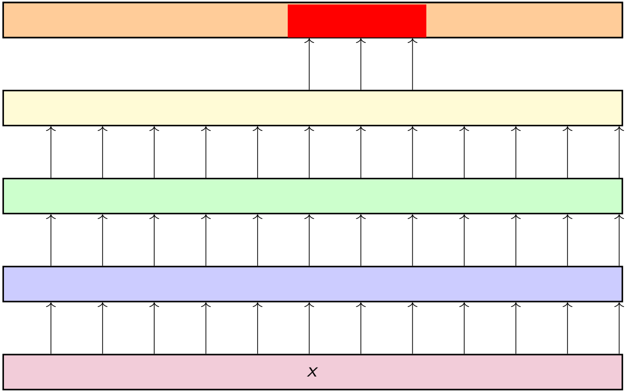

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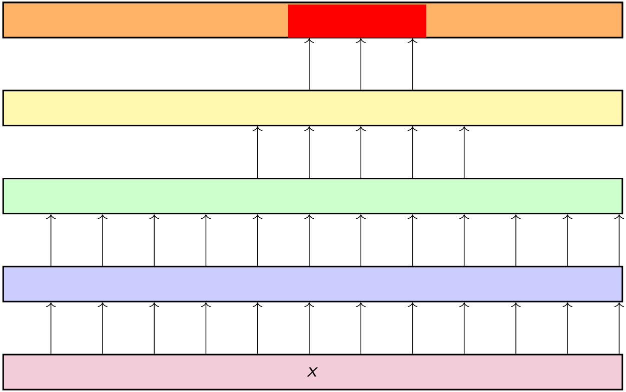

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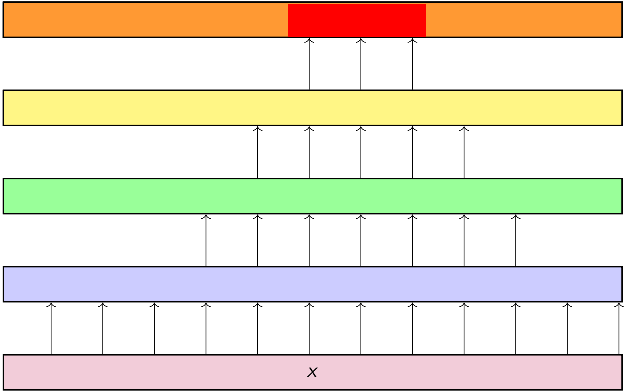

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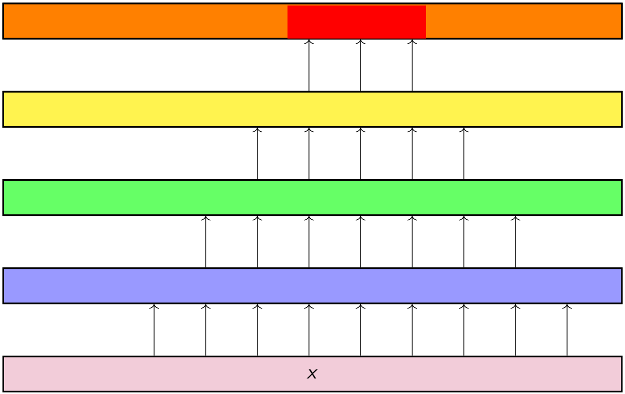

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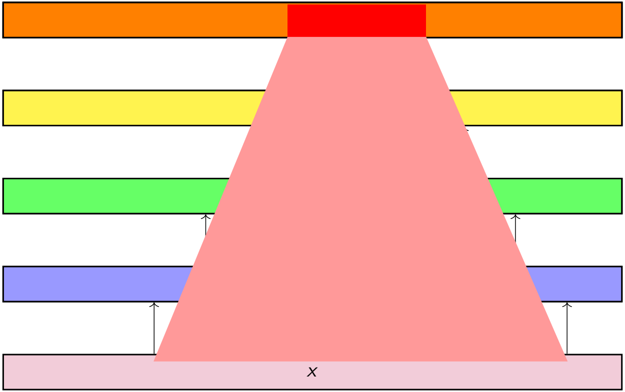

Related results

- Relax frustration-free assumption to negligible frustration.
- Commuting frustration-free stoquastic Hamiltonian is in NP (for any gap)
- “Classical” definition of the problem

Open problems

- Prove/disprove Stoquastic PCP conjecture
- Non-uniform case
  - There are highly frustrated Hamiltonians with no bad strings
  - Frustration comes from incompatibility of amplitudes
    $\sqrt{1-\varepsilon}\,|0\rangle + \sqrt{\varepsilon}\,|1\rangle$ vs. $\sqrt{\varepsilon}\,|0\rangle + \sqrt{1-\varepsilon}\,|1\rangle$
  - Add more tests
    BT has a consistency test, but not clear that it is “local”
- Connections to Hodge theory

Thank you for your attention!