CPR ALGEBRAS

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Abstract. We characterize reflection positivity in the context of parafermion algebras. In order to do this, we introduce the CPR (Canonical Parafermion Relations) $C^*$-algebras, which generalize the CCR/CAR algebras.

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