Projective unitary representations of infinite dimensional Lie groups

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Abstract

In the context of Lie groups G modeled on (not necessarily closed) locally convex spaces, we show that smooth *projective* unitary representations of G correspond to smooth *linear* unitary representatiosn of a central $\mathbb{T}$-extension $\widehat{G}$. The main point here is to prove that the central extension, which is a topological group by construction, carries an appropriate smooth structure. We use these results to show that the projective representations of loop groups and $\mathrm{Diff}(S^1)$ correspond to linear representations of affine Kac-Moody groups and Virasoro groups, respectively.

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